

Signature of the ηNN configurations in coherent π^0 photoproduction on the deuteron

A. Fix^a

Institut für Kernphysik, Johannes Gutenberg-Universität Mainz, D-55099 Mainz, Germany

Received: 29 September 2005 / Revised version: 19 October 2005 /
Published online: 22 November 2005 – © Società Italiana di Fisica / Springer-Verlag 2005
Communicated by V. Vento

Abstract. The photoproduction of a neutral pion on the deuteron is considered in the energy region around the η threshold, where a bump-like structure was observed at very backward pion angles. Different dynamical aspects which may be responsible for this phenomenon are analysed within a theoretical frame which includes intermediate ηNN configurations. The results show, in particular, that a three-body treatment of the ηNN interaction is of special importance.

PACS. 13.60.Le Meson production – 21.45.+v Few-body systems – 25.20.-x Photonnuclear reactions

1 Introduction

In various scattering and production reactions of pions on the deuteron, the cross-sections exhibit a bump at backward angles for energies around the η production threshold. In early discussions of the underlying dynamical mechanisms the main emphasis was put on the interpretation of these anomalies in terms of dibaryon resonances [1].

Later work, however, was mainly focused on a more natural explanation by relating this structure to the threshold effect caused by the opening of the η production channel. In particular, in a recent experiment on coherent π^0 photoproduction such a bump was observed in the cross-section in the backward direction around a lab photon energy $E_\gamma = 700$ MeV [2]. This nontrivial energy dependence was explained in [3], in which the authors conclude that the mechanism, where first an η -meson is produced on one nucleon which then interacts with the second nucleon followed by π^0 production, is responsible for the enhancement around the η threshold.

The purpose of the present paper is to perform a more detailed and extensive investigation of this phenomenon. The main question we address here is the same as in [3], namely what is the dynamics behind this enhancement? Although it might generally be clear that the observed structure is caused by the appearance of an η -meson in one way or another, the question about the underlying mechanism seems to be nontrivial. To be more specific, we would firstly like to note three main points which will be discussed separately:

i) The wide peak in the $\gamma d \rightarrow \pi^0 d$ cross-section can be a direct consequence of the cusp-like structure in the elementary amplitude $\gamma N \rightarrow \pi^0 N$ in the S_{11} channel near the η threshold. The case in point is the strong coupling between the πN and ηN states in the region of the $S_{11}(1535)$ -resonance resulting in a very pronounced cusp in the electric dipole amplitude E_{0+} for pion photoproduction at $E_\gamma \approx 710$ MeV. The latter was also observed in the energy-independent multipole analyses (see, *e.g.*, the energy-independent solution in ref. [4]). Turning now to the process on the nucleon, which is bound in the deuteron, the elementary amplitude is expected to undergo an energy shift and a broadening of its structure due to the Fermi motion. Of crucial importance is the question to which extent this modification could affect the cusp in the elementary E_{0+} multipole and how prominent the $S_{11}(1535)$ -resonance in the reaction on the deuteron could be.

ii) The bump-like structure could also arise from an additional mechanism, appearing when the elementary photoproduction process is embedded into the deuteron. Namely, one could expect that the anomalies are, at least partially, caused by the three-particle unitary cut in the amplitude $\gamma d \rightarrow \pi^0 d$ starting at the energy of the η threshold. This cut arises because of the possibility to exchange a physical η -meson between the S_{11} -resonance excited on one of the nucleons and the second nucleon. The exchange mechanism is characterized by the pole which turns into the cut after the loop integration. Thus, the opening of a new physical channel leads to an additional contribution in the imaginary part of the amplitude reflecting a new inelasticity. Such a picture is typical for coupled channels and, if the corresponding dynamical equations are exactly

^a e-mail: fix@kph.uni-mainz.de

solved, it results in the three-body unitary relation. Because here we take into account only the leading term in the multiple-scattering series, the whole amplitude does not fulfill the unitary relation. However, the ηNN three-body cut appears already at this level, and the question is: how do the amplitudes behave at the branch point?

iii) A sizable attraction in the ηNN system can lead to a strong correlation between all three particles. It is worth noting that in some work the ηNN interaction is predicted to generate even a bound state in the quasideuteron configuration $(J^\pi; T) = (1^-; 0)$. Results provided by more sophisticated models [5, 6] show, however, that the fundamental ηN interaction is likely to be too weak for yielding binding of the ηNN system, so that only a virtual (antibound) state can be generated. In the context of the present discussion two points are relevant: a) Although the pole “recedes” to the nonphysical region, it remains quite close to the zero energy. As a result, the virtual state can strongly influence physical processes involving an η -meson. Indeed, a variety of theoretical calculations and experimental analyses exhibit a strong rise of the η production cross-section just above threshold. This collective effect, in which all three particles participate, is naturally explained within a three-body model [6]. In this sense, the origin of the pole in the amplitude is not of crucial significance, and the existence of an ηNN bound state is not the necessary condition for an anomalous behavior of the η production cross-section. Important is only how far the pole is from the threshold energy. b) Perturbative models, like the first-order rescattering approximation noted in ii), where only the leading-order terms are kept in the multiple-scattering series, are unable to reproduce the real dynamics of the ηNN system in the low-energy region. The most simple explanation is that the corresponding Neumann series converges very slowly near the pole position [7], so that the leading terms turn out to be a bad approximation to the whole series. From the last considerations we conclude that any realistic study of the role of η -mesons in the reaction $\gamma d \rightarrow \pi^0 d$ should be based on a three-body approach to the ηNN system.

Resuming now the qualitative discussion, we would like to note that all mentioned factors can come into play to form the observed characteristic bump in coherent π^0 photoproduction close to the η threshold. Important is the quantitative relation between the different mechanisms, which is the main object of the present paper. In the following we consider all three points separately with special emphasis with respect to their contribution to the resulting cross-section. A brief description of the formal ingredients in sect. 2 is followed by the discussion of our results for the differential cross-section of $\gamma d \rightarrow \pi^0 d$ in sect. 3. The appendix contains the listing of various formulas used for the calculation of the reaction amplitude in the impulse approximation.

2 The formalism

We start the brief description of the basic formal ingredients by presenting in fig. 1 the diagrams which we con-

sider in the theoretical analysis. The three combinations (a), (a) + (b), and (a) + (b) + (c) present three different, successively improved levels of approximation to the reaction amplitude, corresponding to the three points i) through iii) discussed above. We will refer to them as impulse approximation (IA), first-order rescattering approximation, and three-body calculation, respectively. The starting point of our formalism is the impulse approximation which is completely determined by the elementary amplitude for $\gamma N \rightarrow \pi^0 N$ and the deuteron wave function. As elementary amplitude we take the MAID analysis [8] and for the deuteron wave function the parametrization of the Bonn-potential model (OBEPQ-version) [9] with inclusion of the tensor component. The latter is certainly very important in the region of large momentum transfers. The general expression for the amplitude is given in the appendix.

For the additional contributions of the diagrams (b) and (c), we adopt the following simplifications. Firstly, in the pion exchange contribution to the second diagram only the S_{11} -resonance was taken into account neglecting the contribution of other resonances. Although this neglect leads to an underestimation of the pion rescattering effect it should not strongly affect the quality of our results because of the following reasons: i) since rescattering of intermediate pions is not related to the threshold effects it does not contribute to the formation of the peak structure which we discuss here. What we can expect is only a smooth change of the cross-section at backward angles; ii) the role of pion exchange seems to be not very essential. In ref. [10] where this mechanism was calculated more precisely the corresponding effect in the second resonance region is less than 20%.

Secondly, as is indicated in fig. 1, the three-body problem for the ηNN system was solved only for the s -wave state 1S_0 ($J^\pi; T = 0^-; 1$). As is shown in ref. [6] it is the state of lowest orbital momentum which is mostly distorted by the multiple scatterings between particles. Other states with higher orbital momentum can be included perturbatively within the rescattering approximation (diagrams (a)+(b)). In order to avoid double counting we remove from the three-body amplitude those diagrams which possess the same topology as the ones already included in (a) and (b) shown in the same figure.

For the elementary amplitudes appearing in other diagrams, (b) and (c), of fig. 1, we assume that the photoproduction of an η -meson as well as its interaction with nucleons proceeds exclusively via the excitation of the $S_{11}(1535)$ -resonance. As mentioned above, the same ansatz was adopted for pions. According to this assumption the t -matrix of meson-nucleon scattering is given by the conventional isobar model

$$t_{\alpha N \rightarrow \beta N}(\mathbf{p}, \mathbf{p}'; W) = \frac{g_\alpha(\mathbf{p}) g_\beta(\mathbf{p}')}{W - M_0 - \Sigma_\eta - \Sigma_\pi - \Sigma_{\pi\pi}},$$

$$\alpha, \beta \in \{\pi, \eta\}, \quad (1)$$

where W denotes the invariant energy and Σ_α the various self-energy contributions from the αN channels with $\alpha \in \{\pi, \eta, \pi\pi\}$. The t -matrix is determined by the bare

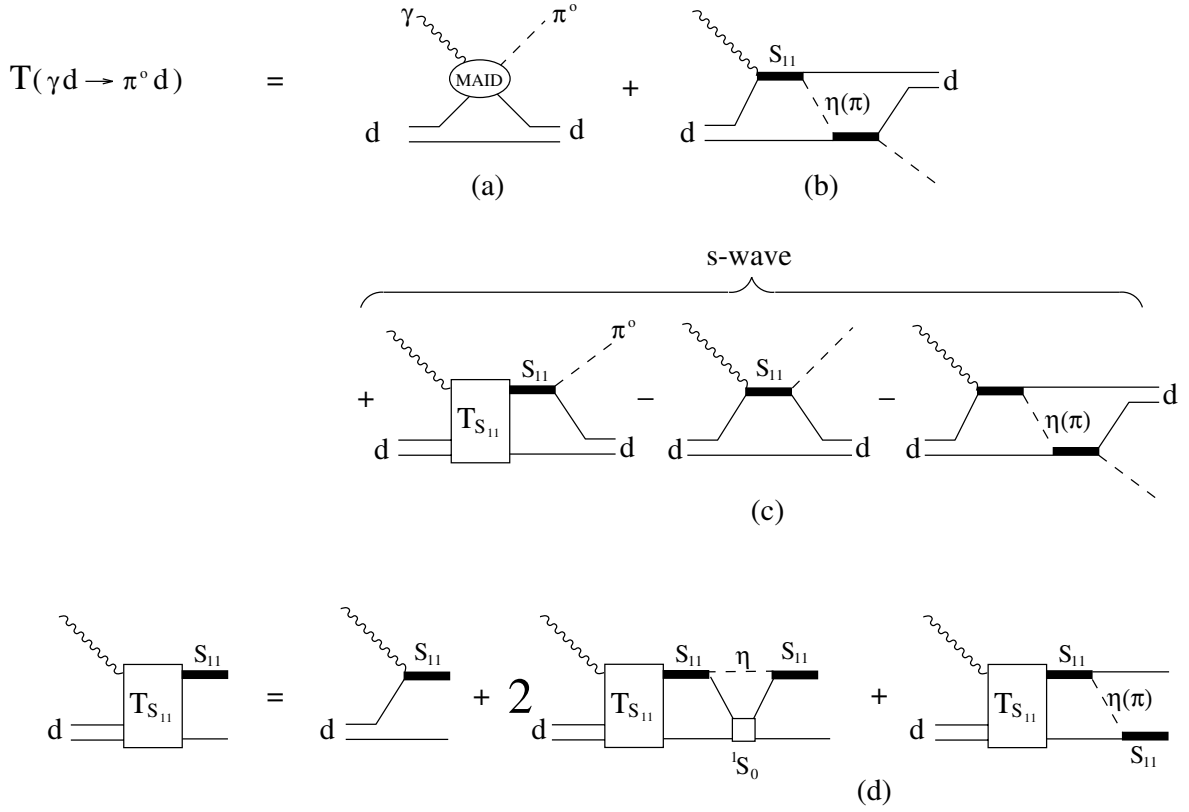


Fig. 1. Diagrams for the reaction $\gamma d \rightarrow \pi^0 d$ included in the present work: (a) impulse approximation; (b) first-order rescattering contribution; (c) additional contribution from three-body dynamics only in the s -wave ($J^\pi; T = 0^-; 1$); (d) equation for the amplitude $T_{S_{11}}$ of photoproduction of the S_{11} -resonance on the deuteron.

resonance mass M_0 and the parameters of the vertex functions g_α , for which we take a simple Hulthén form,

$$g_\alpha(\mathbf{p}) = g_\alpha \left(1 + \frac{p^2}{\Lambda_\alpha^2} \right)^{-1}. \quad (2)$$

The contributions to the self-energy from the various channels are expressed in terms of $g_\alpha(\mathbf{p})$ as

$$\Sigma_\alpha(W) = \frac{1}{2\pi^2} \int_0^\infty \frac{g_\alpha(q)^2}{W - E_N(q) - \omega_\alpha(q) + i\varepsilon} \frac{q^2 dq}{2\omega_\alpha} \quad (3)$$

for $\alpha \in \{\pi, \eta\}$. Since the double-pion channel $\pi\pi N$ is not explicitly included in the calculation, primarily because of its rather weak coupling to the $S_{11}(1535)$, we parametrize, following [11], the corresponding self-energy in a simplified manner as a pure imaginary contribution proportional to the three-particle phase space,

$$\Sigma_{\pi\pi} = -\frac{i}{2} \gamma_{\pi\pi} \frac{W - M_N - 2m_\pi}{m_\pi}. \quad (4)$$

For the photoproduction amplitude we take the same ansatz as in (1) where one hadronic vertex function is replaced by the electromagnetic vertex $g_{\gamma N}$ for $\gamma N \rightarrow S_{11}$ which depends only on the invariant energy W and is

parametrized in the form

$$g_{\gamma p}(W) = \begin{cases} \frac{e}{\sqrt{4\pi}} \sum_{n=0}^4 a_n \left(\frac{q_\pi}{m_\pi} \right)^n, & W \geq M_N + m_\pi, \\ g_{\gamma p}(M_N + m_\pi), & \text{else,} \end{cases} \quad (5)$$

$$g_{\gamma n}(W) = -0.82 g_{\gamma p}(W),$$

with

$$q_\pi = \sqrt{(W^2 - (M_N + m_\pi)^2)(W^2 - (M_N - m_\pi)^2)}/2W. \quad (6)$$

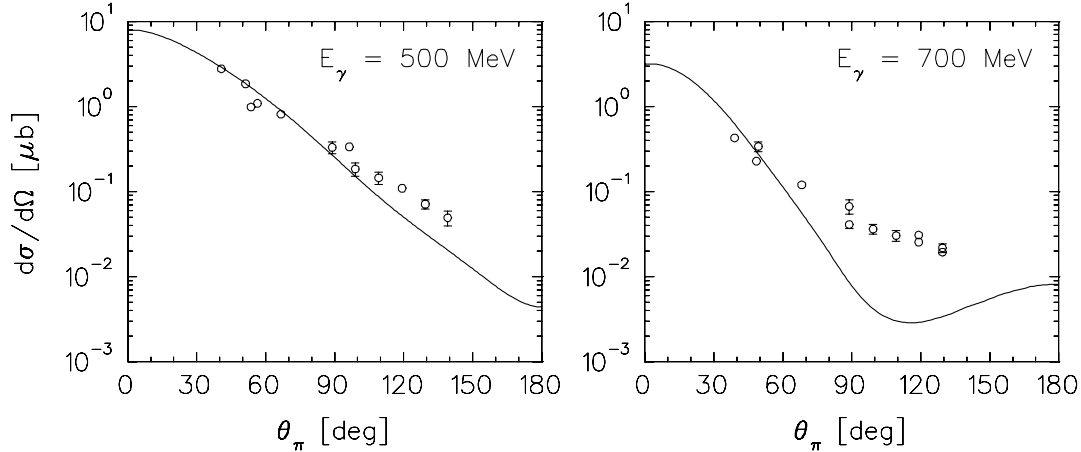
The isospin dependence of the $S_{11}(1535)$ photo-excitation amplitude was taken according to the relation [12]

$$\frac{\sigma(\gamma p \rightarrow \eta p)}{\sigma(\gamma n \rightarrow \eta n)} \approx 0.67. \quad (7)$$

The parameters, appearing in expressions (1) through (5), are listed in table 1. They are chosen in such a way that, on the one hand, the reactions $\gamma N \rightarrow \alpha N$ and $\pi^- p \rightarrow \eta n$ are well reproduced in the S_{11} channel as presented in our previous works [13,14]. On the other hand, the chosen parameter set predicts for the ηN scattering length the value $a_{\eta N} = (0.5 + i0.3)$ fm which we consider as an approximate average of the various values provided by the ηN analyses.

Table 1. Parameters of the ηN scattering matrix in eqs. (1) through (5). The values of Λ_α , $\gamma_{\pi\pi}$, and M_0 are in MeV.

g_η	Λ_η	g_π	Λ_π	$\gamma_{\pi\pi}$	M_0	a_0	a_1	a_2	a_3	a_4
2.00	694.6	2.51	404.5	4.3	1598	$5.502 \cdot 10^{-1}$	$-1.923 \cdot 10^{-2}$	$1.018 \cdot 10^{-1}$	$2.255 \cdot 10^{-3}$	$-7.042 \cdot 10^{-3}$

**Fig. 2.** Angular distribution for $\gamma d \rightarrow \pi^0 d$ at two photon energies. The curves are the results of the impulse approximation (IA). The data are taken from the compilation in [16].

The method which we use to solve the three-body problem for the ηNN interaction is described in ref. [6] and we refer the reader to this paper for the details. Here we only would like to mention that the key point of the method is the separable representation of the driving two-body interaction in the πN , ηN , and NN subsystems. The corresponding t -matrix for meson-nucleon multiple channel scattering is given by the isobar formula (1). For the NN sector, we use the separable representation of the Bonn potential as given in ref. [15] for the 1S_0 and 3S_1 configurations.

As is well known, the separable ansatz makes it possible to reformulate the three-body problem in terms of two-body scattering between quasiparticles. As a consequence, the solution of the problem is given by an amplitude $T_{S_{11}}$ of the effective transition $\gamma d \rightarrow NS_{11}$ as presented in fig. 1(d). The needed physical amplitude $\gamma d \rightarrow \pi^0 d$ is then obtained through an additional loop integration (the first diagram from the left in fig. 1(c)).

3 Results and discussion

In order to demonstrate the quality of the impulse approximation, we present in fig. 2 the resulting pion angular distribution, and compare them with the available data from the compilation of [16]. Although the agreement is quite satisfactory for the forward angles, the data are significantly underestimated in the backward region. We will return to this point at the end of the discussion. A comparison with the IA calculations of ref. [10] also exhibits differences which are, however, not very dramatic. For instance, our cross-section at $E_\gamma = 700$ MeV shows quite a monotonic behavior and does not possess a deep minimum at very forward angles as obtained in ref. [10]. It is

intuitively clear that as long as small angles are considered where the two-nucleon effects are minimal (except for pion two-body absorption), the magnitude of the cross-section should be mostly determined by the elementary amplitude. Therefore, the difference between the two theoretical results has to be ascribed primarily to the differences in the corresponding elementary operators, especially in the spin-flip part (see eq. (A.4)), which accounts for the dominant fraction of the forward cross-section on the deuteron.

Now we turn to our main results on the additional interaction effects as presented in fig. 3. The curves show the predictions according to the different approximations discussed in the introduction. As one might expect, at forward angles the cross-section shows very little influence of these effects. With increasing momentum transfer, corresponding to increasing pion emission angles, small inter-nuclear distances come into play and thus corrections to the simple IA calculation from the two-nucleon mechanisms become more and more important. Furthermore, a bump in the energy dependence around $E_\gamma = 650$ MeV is clearly visible at very backward angles. Among other things, this fact can be considered as strong evidence that primarily the two-nucleon mechanisms are responsible for this phenomenon.

This obvious statement does not, however, diminish the role of the single-nucleon response. As one readily sees in the right panel of fig. 3, some nontrivial structure, a shoulder, appears in the cross-section near the η threshold already in the impulse approximation where the second nucleon is not actively involved. A more detailed analysis reveals, however, that the enhancement is not caused by the presence of the $S_{11}(1535)$ -resonance. Rather it is a combined effect of different terms in the MAID amplitude which we use here.

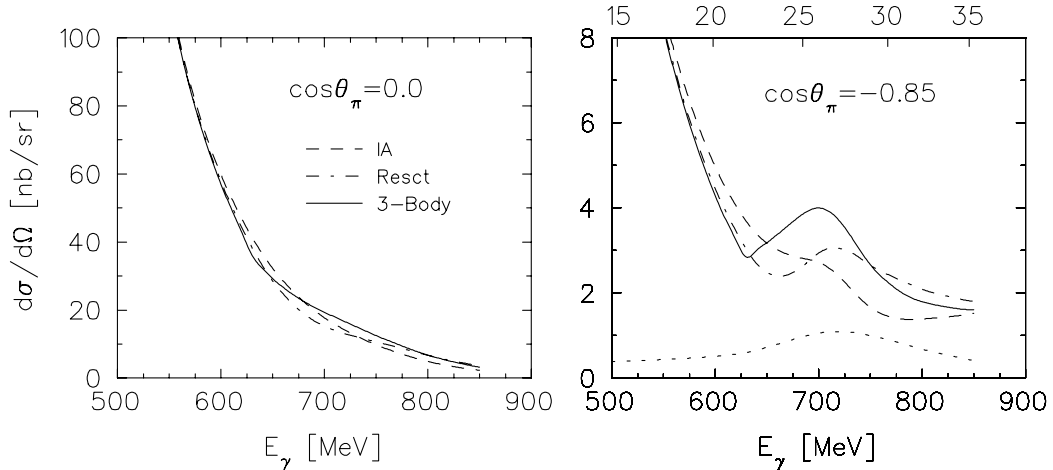


Fig. 3. Differential cross-section for $\gamma d \rightarrow \pi^0 d$ as a function of incident photon energy for two pion emission angles in the γd c.m. system. The curves show the results of the impulse approximation (dashed), first-order rescattering (dash-dotted) and three-body calculation (solid). The dotted curve shows the contribution of the η rescattering term alone (diagram (b) in fig. 1) where the resonance propagators were substituted by constants (see text). The corresponding values of the four-momentum transfer squared in fm^{-2} are shown on the top abscissa.

On the other hand, we would like to note that the $S_{11}(1535)$ -resonance itself does in principle produce a slight shoulder close to the η threshold, which, as already mentioned in the introduction, is a signature of the cusp in the E_{0+} multipole smeared out by the Fermi motion in the deuteron. However, this resonance contributes little to the coherent reaction on the deuteron, so that this cusp-like structure turns out to be invisible in the cross-section. Hence, the slight enhancement observed in the otherwise monotonic behavior of the IA cross-section is not related to the ηNN dynamics and should be ascribed to properties of the elementary pion production amplitude in this region.

Now, when the first-order rescattering of the produced π and η mesons is included, a peak structure clearly evolves as exhibited by the dash-dotted curve in fig. 3. As was emphasized in [3], the main origin of this effect is the presence of the $S_{11}(1535)$ -resonance in the diagram with η rescattering. Our results confirm this statement. It is interesting that the new three-body unitary cut in the amplitude connected with the ηNN channel also makes a slight contribution to the formation of the bump. Indeed, if the strong energy dependence of the amplitudes $\gamma N \rightarrow \eta N$ and $\eta N \rightarrow \pi^0 N$ (see diagram (b) in fig. 1) near the η threshold is “neutralized” by keeping the S_{11} propagator constant, the rescattering term alone exhibits slight enhancement (dotted line in fig. 3). Similar results were presented in ref. [3] where threshold effects in the η rescattering mechanism were analysed (see figs. 5e and f of ref. [3]).

Turning now back to the discussion of three-body effects in π^0 photoproduction, the three-body calculation clearly predicts a much more prominent peak structure accompanied by a slight shift to lower energies. Thus, the difference between the solid and the dash-dotted curves demonstrates convincingly the importance of the higher-order terms in the multiple-scattering series for the intermediate ηNN interaction. It is worth noting that in the

“ideal case” the measurement of the peak might be an indicator of the dynamical properties of the ηNN system. In particular, if a bound ηNN state existed, it would appear in the πNN channel as a pure s -wave three-body resonance. The mass difference $2M_N + m_\eta - M_R$, where M_R is the mass of this hypothetical resonance, would then give the binding energy. If, on the other hand, the ηNN scattering amplitude possesses only a virtual pole, as is predicted by our model, the resonance peak should fall directly on the η threshold. Nevertheless, it is obvious that the extraction of such information would require a quite thorough partial-wave analysis (we need to consider only the s -wave contribution) and, moreover, a very precise energy resolution and a small statistical error, which could then allow one to fix the resonance position. A similar scheme was already realized in the experiments aimed at a search for η mesic nuclei with ${}^3\text{He}$ [17] and heavier targets [18]. But although the quality of the data permits one to make quite definite conclusions about the peak position, there are still problems with the interpretation of the measured angular distribution and the energy dependence of the corresponding cross-section within existing theoretical models [17].

A comparison of our results in fig. 4 with preliminary data from ref. [2] shows that the theory underestimates the observed cross-section at backward angles by about one order of magnitude. The same discrepancy is noted for the results of ref. [3] where the cross-section is also far below the data in the same region. Yet, the authors of [3] use an oversimplified operator for $\gamma N \rightarrow \pi N$ and it is therefore difficult to identify the prime reason of this drawback. Furthermore, similar underestimation may be exhibited in the work of ref. [10], although the deviation is not as significant as in our case.

The noted discrepancy might not be very surprising after all. We can expect that as long as the pions are produced on the individual nucleons and as long as the angular dependence of the elementary amplitude $\gamma N \rightarrow \pi^0 N$

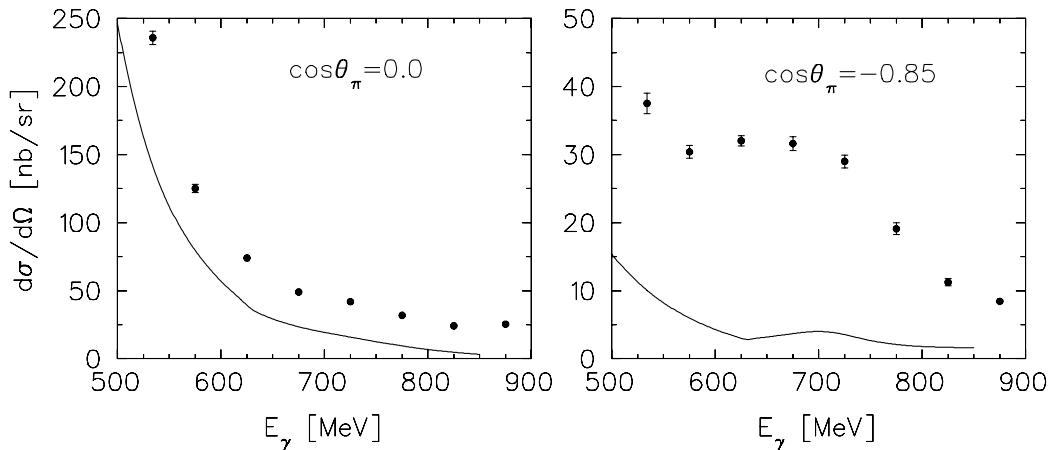


Fig. 4. Comparison of our three-body results (solid line in fig. 3) with CLAS preliminary data [2].

is not varying strongly, the form of the differential cross-section is mainly governed by the deuteron form factor. In order to locate the appropriate portion of the form factor which enters in the kinematical conditions, we present on the top x -axis in fig. 3 the corresponding four-momentum transfer squared. As one can see, its characteristic values range between 15 and 35 fm⁻², where the form factor as seen in electron scattering exhibits a sizeable sensitivity to higher-order mechanisms like π and ρ MECs (see, *e.g.*, ref. [19]).

Therefore, it is very probable that additional two-nucleon mechanisms, not included in the present calculation, will make sizeable contributions to the backward cross-section. For example, meson exchange currents were found to be quite significant for π^+ photoproduction on ^3He in ref. [20]. In this case, the effect is associated with double-meson photoproduction followed by absorption of one of the mesons on the spectator nucleon. However, if such mechanisms are really quite important, their inclusion would tend to diminish the effect discussed above, *i.e.* the relative size of the peak as its manifestation. In this case, we will have to face again the same question about the origin of the structure observed in the π^0 photoproduction.

4 Conclusion

We have discussed various aspects of the influence of η degrees of freedom on coherent π^0 photoproduction on the deuteron. Two interrelated features determine the significance of the η -meson in this reaction. Firstly, the strong coupling between πN and ηN states in the energy region of the $S_{11}(1535)$ -resonance leads to a significant admixture of the ηNN configuration to the NS_{11} intermediate states. What is more important, in contrast to pions, slow η -mesons interact strongly with nearby nucleons. As predicted in a variety of investigations, this dynamical feature leads to highly correlated ηNN states, which should manifest themselves in the s -wave part of the π^0 photoproduction amplitude.

Both factors lead to the appearance of a pronounced bump in the energy dependence of the backward differential cross-section around the η production threshold. This effect was analysed in ref. [3] on the basis of a theoretical model where in addition to the mere impulse approximation π and η rescatterings were included. In contrast to the conclusion of [3], we find that already in the impulse approximation a shoulder appears in the cross-section around $E_\gamma = 700$ MeV. However, as is explained in the present work, this effect does not have a deep physical significance, and is likely to reflect the special structure of the pion production amplitude used here.

The inclusion of first-order rescattering and, finally, all terms in the multiple-scattering series within a three-body model shifts the peak position and makes it significantly more pronounced. In particular, it was shown that a three-body treatment of the ηNN interaction is of special importance for the understanding of the reaction dynamics.

In general, according to our results, the physics behind the bump structure in the cross-section for $\gamma d \rightarrow \pi^0 d$ may be much more complicated than was presented in ref. [3]. Among other things, we are very sceptical about the possibility to extract model-independent information on the fundamental ηN interaction from the (γ, π^0) reactions.

The present calculation as well as those presented in ref. [3] can be considered as a natural explanation of the experimental results reported in [2]. However, the status of these findings is quite unclear, because the theory strongly underestimates the data in the relevant angular region, making any quantitative analysis of the observed cross-section impossible. Further theoretical investigations in this field are certainly needed.

The author would like to thank Hartmuth Arenhövel for useful discussions and a critical reading of the manuscript. This work was supported by the Deutsche Forschungsgemeinschaft (SFB 443).

Appendix A. Impulse approximation for the amplitude $\gamma d \rightarrow \pi^0 d$

Here we give a brief outline of the impulse approximation of the T -matrix of coherent pion photoproduction on the deuteron in the c.m. frame,

$$\gamma(\omega_\gamma, \mathbf{k}, \lambda) + d(E_d, -\mathbf{k}) \rightarrow \pi(\omega_\pi, \mathbf{q}) + d(E'_d, -\mathbf{q}), \quad (\text{A.1})$$

where energy and momenta of the participating particles are given in parenthesis, and λ stands for the circular photon polarization index. In the impulse approximation, the amplitude $T_{mm'\lambda}$ for the transition between the target states with spin projections m and m' on the z -axis, chosen along the photon momentum, reads

$$T_{mm'\lambda} = 2 \int \frac{d^3 p}{(2\pi)^3} \phi_{m'}^\dagger(\mathbf{p}') t_{\pi\gamma}^\lambda(\mathbf{k}, \mathbf{p}_i, \mathbf{q}, \mathbf{p}_f) \phi_m(\mathbf{p}), \quad (\text{A.2})$$

with $t_{\pi\gamma}^\lambda$ standing for the corresponding elementary amplitude $\gamma N \rightarrow \pi^0 N$. Furthermore, the vectors \mathbf{p}_i and \mathbf{p}_f denote initial and final momenta of the active nucleon in the deuteron, for which we have $\mathbf{p}_i = \mathbf{p} - \mathbf{k}/2$ and $\mathbf{p}_f = \mathbf{p} - \mathbf{q} + \mathbf{k}/2$, and $\mathbf{p}' = \mathbf{p} + (\mathbf{k} - \mathbf{q})/2$ denotes the relative momentum in the final deuteron state.

For the deuteron wave function we use the familiar ansatz

$$\phi_m(\mathbf{p}) = \sum_{L=0,2} \sum_{m_L m_S} (L m_L 1 m_S | 1 m) u_L(\mathbf{p}) Y_{L m_L}(\hat{\mathbf{p}}) \chi_{m_S} \zeta_0, \quad (\text{A.3})$$

where the last two terms denote spin and isospin wave functions, respectively.

For a nuclear application it is convenient to split the amplitude $t_{\pi\gamma}$ into spin-independent and spin-flip parts (the index λ is omitted for convenience in the following expressions),

$$t_{\pi\gamma} = K + i\mathbf{L} \cdot \boldsymbol{\sigma}. \quad (\text{A.4})$$

Then, using standard angular-momentum algebra, the reaction amplitude (A.2) can be put into the following form:

$$\begin{aligned} T_{mm'} &= A\sqrt{3} \sum_{\Lambda=0}^2 (-1)^{M_\Lambda} \sqrt{2\Lambda+1} (1m\Lambda - M_\Lambda | 1m') \\ &\times \sum_{LL'=0,2} \int \frac{d^3 p}{(2\pi)^3} \left[\begin{Bmatrix} L & 1 & 1 \\ 1 & L' & \Lambda \end{Bmatrix} \{Y^{[L']}(\hat{\mathbf{p}}') \otimes Y^{[L]}(\hat{\mathbf{p}})\}_{M_\Lambda}^{[A]} K \right. \\ &- (-1)^\Lambda \sqrt{6} \sum_{l=0}^3 \sqrt{2l+1} \begin{Bmatrix} \Lambda & 1 & 1 \\ 1 & 1 & 1 \\ l & L & L' \end{Bmatrix} \\ &\left. \times \{ \{Y^{[L']}(\hat{\mathbf{p}}') \otimes Y^{[L]}(\hat{\mathbf{p}})\}^{[l]} \otimes L^{[1]} \}_{M_\Lambda}^{[A]} \right] u_{L'}(\mathbf{p}') u_L(\mathbf{p}). \end{aligned} \quad (\text{A.5})$$

As for the isospin structure, it is easy to understand that from all three amplitudes in the isospin decomposition of the elementary operator for pion photoproduction with Cartesian index $\alpha = 1, 2, 3$ [21]

$$t_{\pi\gamma} = M^{(0)}\tau_\alpha + M^{(-)}\frac{1}{2}[\tau_\alpha, \tau_3] + M^{(+)}\delta_{\alpha 3}, \quad (\text{A.6})$$

only $M^{(+)}$ can contribute to the coherent process on the deuteron.

Using standard normalization of particle states, the cross-section related to the same c.m. frame reads

$$\frac{d\sigma}{d\Omega} = \frac{q}{\omega_\gamma} \frac{E_d E'_d}{(4\pi W)^2} \frac{1}{6} \sum_{mm'\lambda} |T_{mm'\lambda}|^2. \quad (\text{A.7})$$

References

1. M. Akemoto, K. Baba, I. Endo *et al.*, Phys. Rev. Lett. **50**, 400 (1983).
2. Y. Ilieva (for the CLAS collaboration), *Proceedings of the 17th International IUPAP Conference on Few-Body Problems in Physics, Durham NC (2003)*, Nucl. Phys. A **737**, S158 (2004), nucl-ex/0309017.
3. A.E. Kudryavtsev, V.E. Tarasov, I.I. Strakovsky *et al.*, Phys. Rev. C **71**, 035202 (2005).
4. R.A. Arndt, R.L. Workman, Z. Li, L.D. Roper, Phys. Rev. C **42**, 1853 (1990).
5. S. Wycech, A.M. Green, Nucl. Phys. A **663-664**, 529c (2000).
6. A. Fix, H. Arenhövel, Nucl. Phys. A **697**, 277 (2002).
7. E.W. Schmid, H. Ziegelman, *The Quantum Mechanical Three-Body Problem* (Pergamon, New York, 1974).
8. D. Drechsel, S.S. Kamalov, L. Tiator, Nucl. Phys. A **645**, 145 (1999).
9. R. Machleidt, K. Holinde, Ch. Elster, Phys. Rep. **149**, 1 (1987).
10. H. Garcilazo, E. Moya de Guerra, Phys. Rev. C **52**, 49 (1995).
11. C. Bennhold, H. Tanabe, Nucl. Phys. A **530**, 62 (1991).
12. V. Hejny, P. Achenbach, J. Ahrens *et al.*, Eur. Phys. J. A **6**, 83 (1999).
13. A. Fix, H. Arenhövel, Eur. Phys. J. A **19**, 275 (2004).
14. A. Fix, H. Arenhövel, Phys. Rev. C **68**, 044002 (2003).
15. J. Haidenbauer, Y. Koike, W. Plessas, Phys. Rev. C **33**, 439 (1986).
16. K. Ukai, T. Nakamura, INS University of Tokyo, Report INS-TEC-22, 1985.
17. M. Pfeiffer, J. Ahrens, J.R.M. Annand *et al.*, Phys. Rev. Lett. **92**, 252001 (2004).
18. G.A. Sokol, A.I. Lvov, L.N. Pavlyuchenko, nucl-ex/0106005.
19. H. Arenhövel, F. Ritz, T. Wilbois, Phys. Rev. C **61**, 034002 (2000).
20. J.A. Gomez Tejedor, S.S. Kamalov, E. Oset, Phys. Rev. C **54**, 3160 (1996).
21. G.F. Chew, M.L. Goldberger, F.E. Low, Y. Nambu, Phys. Rev. **106**, 1345 (1957).